CHARGES AS INTEGRALS OF DENSITIES

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It is shown that unitary operators giving rise to approximate symmetries associated with nonconserved currents cannot exist in relativistic field theory.

In recent literature a number of difficulties connected with the introduction of space integrals of the zero component of a nonconserved local current have been pointed out.[1-3] In this context it is interesting to remark that some of the arguments in those papers rested upon the assumption that

$$Q(t)=\int j^0(\vec{x},t)d^3x=\lim_{v\to\infty}\int_v j^0(\vec{x},t)d^3x \tag{1}$$

should be understood in the sense of a weak limit, i.e.,

$$(\psi,Q(t)\varphi)=\lim_{v\to\infty}(\psi,\int_v j^0(\vec{x},t)d^3x\,\varphi). \tag{2}$$

This is, however, too strong an assumption as can be seen clearly already in the case of conserved current and exact symmetry. In fact, taking the very trivial example of a free scalar charged field one can build a two-particle state

$$|2\rangle=\int f(\vec{k}_1,\vec{k}_2)\frac{(\vec{k}_1+\vec{k}_2)\cdot(\vec{k}_1-\vec{k}_2)}{(\vec{k}_1+\vec{k}_2)^2}\times a^+(\vec{k}_1)b^+(\vec{k}_2)d^3k_1d^3k_2|0\rangle, \tag{3}$$

and choosing $f(\vec{k}_1,\vec{k}_2)$ such that

$$f(\vec{k}_1,\vec{k}_2)>0,\quad \langle 2|2\rangle=1, \tag{4}$$

one arrives at

$$\langle 2|Q|0\rangle=0\neq\lim_{v\to\infty}\langle 2|\int_v\rho(\vec{x},0)d^3x|0\rangle, \tag{5}$$

where Q and ρ are the well-known charge and charge density operators in a free theory.

A general analysis[4] shows that even in the case of exact symmetries, Eq. (2) will hold only for a particular class of states which have the physical meaning of differing from the vacuum only in a finite region.

It is therefore desirable to obtain a theorem of Coleman[1] and Fabri-Picasso[2] type without any use of Eq. (2). We take Eq. (1) as only a heuristic guide and assume the following:
(A) There exist unitary operators $U(\tau,t)=\exp[i\tau Q(t)]$ that transform local operators into local operators (which might belong to different space-time regions):

$$U(\tau,t)AU^{-1}(\tau,t)=A_{\tau,t}. \tag{6}$$

(B) These operators satisfy

$$[P_i,U(\tau,t)]=0\quad(i=1,2,3). \tag{7}$$

(C) The transformed operators satisfy

$$(d/d\tau)A_{\tau,t}=i[Q(t),A_{\tau,t}]\equiv i[\int_v j^0(\vec{x},t)d^3x,A_{\tau,t}], \tag{8}$$

where because of local commutativity the integration is only taken over a finite region;[4] it is this relation which replaces the formal Eq. (1).

We can work without loss of generality with a unique vacuum state, which is equivalent to the assumption of the irreducibility of the algebra of local operators.[5,6]

From (7) one obtains

$$P_iU(\tau,t)|0\rangle=0\quad(i=1,2,3) \tag{9}$$

and, therefore, due to the uniqueness of the vacuum (only discrete eigenstate of the linear momentum),

$$U(\tau,t)|0\rangle=|0\rangle\exp[i\tau\Lambda(t)], \tag{10}$$

and by a trivial redefinition of $Q(t)$ one arrives at

$$U(\tau,t)|0\rangle=|0\rangle. \tag{11}$$

Equation (11) implies the following invariance property of the Wightman functions:

$$\langle 0|A_{\tau,t}|0\rangle=\langle 0|A|0\rangle, \tag{12}$$

where A is any local operator and, in particular, could be a local Wightman polynomial in the basic fields.[5]

We shall show now that $A_{\tau,t}$ is independent of t to arrive at the conclusion that there exists an exact symmetry of the theory in the usual sense. For this purpose we introduce

$$\varphi(x)=\partial j^{\mu}(x)/\partial x^{\mu},\quad \pi(x)=\partial\varphi(x)/\partial t \tag{13}$$

and find, using (8) and (12),

$$\frac{d}{d\tau}\langle 0|\pi_{\tau,t}{}^{(0)}|0\rangle\Big|_{\tau=0}$$

$$=0=i\langle 0|[\int_V j^0(\vec{x},t)d^3x,\pi(0)]|0\rangle, \tag{14}$$

and therefore, since boundary terms drop because of local commutativity,

$$\langle 0|[\int_V\varphi(\vec{x},0)d^3x,\pi(0)]|0\rangle=0, \tag{15}$$

which using the Lehmann-Källen representation[7,8]

$$\langle 0|[\varphi(x),\varphi(y)]|0\rangle=\int_0^\infty\rho(\kappa^2)\Delta(x-y,\kappa^2)d\kappa^2,$$

$$\rho(\kappa^2)\geq 0, \tag{16}$$

implies

$$\int_0^\infty\rho(\kappa^2)d\kappa^2=0 \tag{17a}$$

and, therefore,

$$\rho(\kappa^2)=0. \tag{17b}$$

With (17b) one has that

$$\langle 0|\varphi(x)\varphi(y)|0\rangle=\int_0^\infty\rho(\kappa^2)\Delta^{(+)}(x-y,\kappa^2)d\kappa^2=0, \tag{18}$$

and since the metric in the Hilbert space is positive definite, Eq. (18) gives

$$\varphi(x)|0\rangle=0. \tag{19}$$

and by the Johnson-Federbush[9] theorem,

$$\varphi(x)=\partial j^{\mu}/\partial x^{\mu}=0. \tag{20}$$

From Eq. (20) one deduces now

$$[Q(t)-Q(0),A]\ \text{for all}\ A, \tag{21}$$

and Eq. (21) together with the irreducibility of the algebra of local operators and (11) implies

$$Q(t)=Q(0). \tag{22}$$

Therefore, assumptions (A), (B), and (C) imply the existence of an exact symmetry which commutes with the space-time translations. It is thus impossible to set up an algebra for the "generators" of approximate symmetries in the sense of Gell-Mann,[10,11] since those generators do not exist. However, the algebra of the currents integrated over a finite but arbitrarily large volume might exist and lead to the same consequences as the formal Gell-Mann algebra.[12]

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[12]My attention was called to the fact that this conjecture has been proved true by B. Schroer and P. Stichel, to be published.

POLARIZATION IN PION-PROTON SCATTERING FROM 670 TO 3750 MeV/c*

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Using a polarized proton target, we have measured the polarization parameter $P(\theta)$ in pion-proton scattering for both positive and negative pions. Because there seems to be a great deal of current interest in the analysis of pion-proton scattering, we wish to present these experimental results at this time, even though we have not yet completed their analysis. The measurement consisted of scattering pions from polarized target protons and observing the asymmetry in scattered intensity, $I(\theta)$, as the spin directions of the target protons were reversed. The intensity for scattering from a target of polarization, P_T, is

$$I(\theta)_{\text{pol}}=I(\theta)_{\text{unpol}}[1+P(\theta)P_T],$$